Ganit Prabhutva – Level II (2025-2026) – 8th

Sr. No.	Solution	Total Marks
Q.1.	$\sqrt{1+2025\sqrt{1+2026\sqrt{1+2027*2029}}}$ $\sqrt{1+2025\sqrt{1+2026\sqrt{1+(2028+1)(2028-1)}}}$ $\sqrt{1+2025\sqrt{1+2026\sqrt{1+2028^2-1}}}$ $\sqrt{1+2025\sqrt{1+2026*2028}}$ $\sqrt{1+2025\sqrt{1+(2027+1)(2027-1)}}$ $\sqrt{1+2025\sqrt{1+2027^2-1}}$ $\sqrt{1+2025*2027}$ $\sqrt{1+(2026+1)(2026-1)}$ $\sqrt{1+2026^2-1}=2026$	5 M
Q.2.	$\sqrt[3]{p(p^2-3p+3)-1}*\frac{p^2-39p-40}{p^2-1}$ $=\sqrt[3]{p^3-3p^2+3p-1}*\frac{(p-40)(p+1)}{(p+1)(p-1)}$ $=\sqrt[3]{(p-1)^3}*\frac{(p-40)}{(p-1)}$ $=(p-1)*\frac{(p-40)}{(p-1)}$ $=\ (p-40)=1000$	5 M

Q.3.	$Let\ the\ sides\ of\ the\ sqaures\ be\ x, (x+2), (x+4)\ \&\ (x+6)$ $Area\ of\ shaded\ region$ $= [(x+2)^2 - x^2] + [(x+6)^2 - (x+4)\text{^}2]$ $\therefore 64 = x^2 + 4x + 4 - x^2 + x^2 + 12x + 36 - x^2 - 8x - 16$ $\therefore 64 = 8x + 24$ $\therefore x = 5\ units$ $\therefore Area \frac{b}{w} largest\ and\ smallest\ square = (x+6)^2 - x^2$ $= 96\ sq. units$	6 M
Q.4.	$x\ divided\ by\ 3\ leaves\ a\ remainder\ 2\ means\ (x$ $+ 1) is\ a\ multiple\ of\ 3$ $x\ divided\ by\ 5\ leaves\ a\ remainder\ 3\ means\ (x$ $+ 2)\ is\ a\ multiple\ of\ 5$ $\therefore (x+1)\ \&\ (x+2)\ are\ consecutive\ nos.$ $that\ are\ multiples\ of\ 3\ \&\ 5\ respectively$ $\therefore (x+1) = 9\ \&\ (x+2) = 10\ i.e. x = 8$ $Thus, 8\ is\ smallest\ no. which\ satisfies\ the\ given\ conditions.$ $Thus, 8\ added\ to\ any\ common\ multiple\ of\ 3\ \&\ 5\ will$ $also\ result\ in\ similar\ nos.$ $Thus, the\ required\ nos. are\ 450 \le 15k + 8 \le 550$ $Thus, the\ required\ nos. are\ 458, 473, 488, 503, 518, 533, 548$	6 M

Q.5.	$Sum\ of\ interior\ angles\ of\ n - sided\ polygon$ $= (n-2) * 180$ $\therefore Sum\ of\ interior\ angles\ of\ a\ pentagon = (5-2) * 180$ $= 540$ $Let\ the\ interior\ angles\ be\ 3x, 4x, 5x, 6x, 9x$ $\therefore 3x + 4x + 5x + 6x + 9x = 540$ $\therefore x = 20$ $i.e. the\ angles\ are\ 60, 80, 100, 120\ \&\ 180$ $But, interior\ angle \neq 180$ $Hence, not\ poosible$	6 M
Q.6.	$\frac{n^3 + 9}{n+3}$ $\frac{(n^3 + 27) - 18}{n+3}$ $= \frac{(n+3)(n^2 + 9 - 3n) - 18}{n+3}$ $= (n^2 + 9 - 3n) - \frac{18}{n+3}$ $Thus, for\ complete\ division, (n+3)\ must\ be\ a\ factor\ of\ 18$ $Thus, (n+3) can\ be\ either \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$ $Thus, n \in \{-21, -12, -9, -6, -5, -4, -2, -1, 0, 3, 6, 15\}$	6M

Q.7. (7 M)

$$\frac{a+b+d}{c}+1+\frac{b+c+d}{a}+1+\frac{a+c+d}{b}+1+\frac{a+b+c}{d}+1-4$$

$$=\frac{a+b+d+c}{c}+\frac{b+c+d+a}{a}+\frac{a+c+d+b}{b}+\frac{a+b+c+d}{d}-4$$

$$=(a+b+c+d)\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}+\frac{1}{d}\right)-4$$

$$=\frac{(a+b+c+d)(abd+bcd+acd+abc)}{abcd}-4$$

$$=(a+b+c+d)*\frac{0}{abcd}-4$$

$$=-4$$

Q.8. (7 M)

$Let\ us\ assume\ A > B$

$AB = 10A + B\ and\ BA = 10B + A$

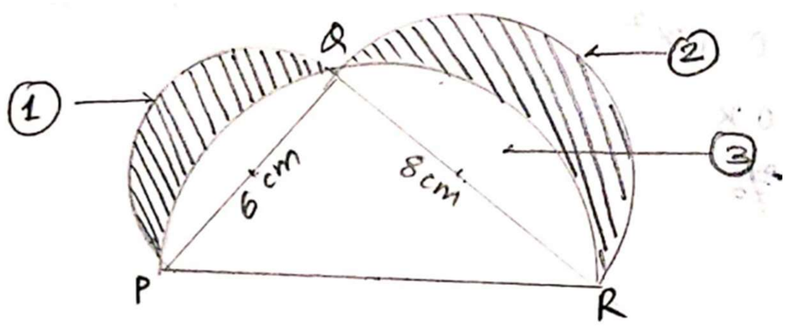
$AB - BA = 10A + B - 10B - A = 9(A - B)$

$\therefore\ digits\ A\ \&\ B\ should\ be\ such\ that$

$9(A - B) must\ be\ a\ square\ of\ an\ integer$

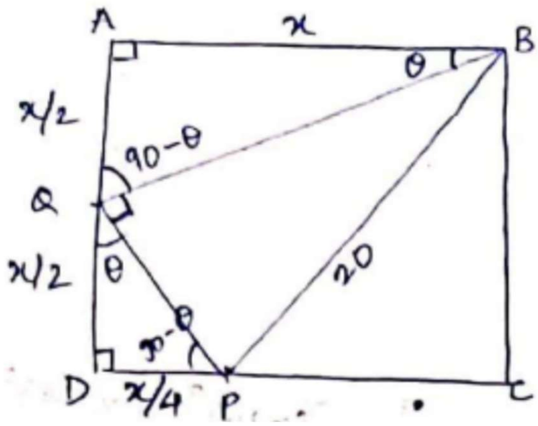
$\therefore (A - B) = 0\ or\ 1\ or\ 4\ or\ 9$

0,0	1,0	4,0	9,0
1,1	2,1	5,1	
2,2	3,2	6,2	
3,3	4,3	7,3	
4,4	5,4	8,4	
5,5	6,5	9,5	
6,6	7,6		
7,7	8,7		
8,8	9,8		
9,9			

Q.9.	$264 = 8 * 3 * 11$ $\therefore$ *the required six* $-$ *digit number must be divisible by* $3, 8$ & 11 *Divisibility by* $3 \rightarrow (1+2+3+3+4+5) = 18$ *Divisibility by* 11 $\rightarrow (2,4,3)$ & $(1,3,5)$ *must appear at alternate places* *Divisibility at* $8 \rightarrow$ *Last* $3 -$ *digits must be* $312, 432$ *or* 352 $\therefore$ *Possible numbers are as below*: 345312, 543312, 135432, 531432, 341352, 143352	7 M
Q.10.	 $In\ \Delta PQR \rightarrow PQ^2 + QR^2 = PR^2$ $\therefore 6^2 + 8^2 = PR^2$ $\therefore PR = 10\ cm$ *Area of shaded region* $=$ $\{$*Area of* $1st$ *semicircle* $+$ *Area of* $2nd$ *semicircle* $+$ *Area of* $\Delta PQR -$ *Area of* $3rd$ *semicicle*$\}$ $= \pi * \frac{3^2}{2} + \pi * \frac{4^2}{2} + \frac{1}{2} * 6 * 8 - \pi * \frac{5^2}{2}$ $= 24\ cm^2$	7 M

Q.11.	$For\ trader\ A \rightarrow CP = Rs\ 1000$ $\therefore SP = 1000\left(1+\frac{x}{100}\right) = 1000 + 10x$ $\therefore profit = SP - CP = 10x$ $For\ trader\ B \rightarrow CP = Rs\ 2000$ $\therefore MP = 2000\left(1+\frac{2x}{100}\right) = 2000 + 40x$ $\therefore SP = MP\left(1-\frac{x}{100}\right) = (2000+40x)\left(1-\frac{x}{100}\right)$ $\therefore SP = 2000 + 20x - 0.4x^2$ $\therefore Profit = 20x - 0.4x^2$ $Profit\ of\ A = Profit\ of\ B$ $\therefore 10x = 20x - 0.4x^2$ $\therefore x = 25$	7 M
Q.12.	$Case\ 1: Bases\ are\ same, thus\ exponents\ must\ be\ equal$ $\therefore x^2 + 4 = 4x$ $\therefore (x-2)^2 = 0$ $\therefore x = 2$ $Case\ 2: For\ any\ value\ of\ a, 1^a = 1$ $\therefore if\ (x-5) = 1 \rightarrow (x-5)^{x^2+4} = (x-5)^{4x}\ will\ be\ true.$ $\therefore x = 6$ $Case\ 3: For\ positive\ values\ of\ a, 0^a = 0$ $\therefore if\ (x-5) = 0 \rightarrow x = 5$ $\therefore x^2 + 4 = 29\ \&\ 4x = 20$ $\therefore (x-5)^{x^2+4} = (x-5)^{4x}$ $\therefore 0^{29} = 0^{20}$ $\therefore x = 5$ $Case\ 4:$ $-1^{any\ even\ no.} = -1^{any\ othe\ \ even\ no.}\ OR\ -1^{any\ odd\ no.}$ $= -1^{any\ other\ odd\ no.}$ $if\ x = 4, (x-5) = -1\ \&\ x^2 + 4 = 20\ \&\ 4x = 16$ $\therefore (x-5)^{x^2+4} = (x-5)^{4x}$ $\therefore -1^{20} = -1^{16}$ $\therefore x = 4$	7 M

Q.13. **8 M**



$\angle AQB = 90 - \theta \ldots (sum\ of\ angles\ of\ a\ triangle)$

$\angle AQB + \angle BQP + \angle DQP = 180 \ldots (angles\ in\ linear\ pair)$

$\therefore (90 - \theta) + 90 + \angle DQP = 180$

$\therefore \angle DQP = \theta$

$\therefore \angle DPQ = 90 - \theta \ldots (sum\ of\ angles\ of\ a\ triangle)$

$In\ \Delta AQB\ \&\ \Delta DPQ,$

$\angle A \cong \angle D \ldots (\blacksquare ABCD\ is\ a\ square)$

$\angle B \cong \angle Q = \theta \ldots (proved\ above)$

$\angle Q \cong \angle P = 90 - \theta \ldots (proved\ above)$

$\therefore \Delta AQB \sim \Delta DPQ \ldots (A - A\ test\ of\ similarity)$

$\therefore \frac{AB}{AQ} = \frac{DQ}{DP} \qquad \frac{x}{\left(\frac{x}{2}\right)} = \frac{\left(\frac{x}{2}\right)}{DP}$

$\therefore DP = \frac{x}{4} \qquad \therefore PC = \frac{3x}{4}$

$In\ \Delta BPC,$

$BC^2 + PC^2 = BP^2 \qquad \therefore x^2 + \left(\frac{3x}{4}\right)^2 = 20^2 \qquad \therefore x = 16\ cm$

$In\ \Delta QDP,$

$QD^2 + DP^2 = QP^2 \qquad \therefore \frac{x^2}{2} + \left(\frac{x}{4}\right)^2 = QP^2 \qquad \therefore QP = 4\sqrt{5}\ cm$

$In\ \Delta BQP,$

$BQ^2 + QP^2 = BP^2 \qquad \therefore BQ^2 = 400 - 80 \qquad \therefore BQ = 8\sqrt{5}\ cm$

$$Area\ of\ \Delta BQP = \frac{1}{2} * BQ * QP$$

$= \frac{1}{2} * 8\sqrt{5} * 4\sqrt{5} \qquad = 80\ cm^2$

Q.15.	$cycle\ time\ of\ one\ launch\ for\ a\ round\ trip =$ $(Time\ from\ A\ to\ B) + (Halt\ time\ at\ B) +$ $(Time\ from\ B\ to\ A) + (Halt\ time\ at\ A)$	8 M

$\therefore cycle\ time = 60 + 20 + 120 + 20 = 220\ mins$

∴ If we have sufficient launches for the initial 220 mins, we can reuse them for the rest of the service.

$$\therefore Minimum\ Number\ of\ launches\ required = \frac{cycle\ time}{frequency}$$

$$= \frac{220\ min}{20\ min} = 11$$

Time Stamp	Halt at A	A	B	Halt at B
06:00		1	2	
06:20		3	4	
06:40		5	6	
07:00		7	8	1
07:20		9	1	3
07:40		10	3	5
08:00	2	11	5	7
08:20	4	2	7	9
08:40	6	4	9	10
09:00	8	6	10	11
09:20	1	8	11	2
09:40	3	1	2	4

$If\ a\ launch\ starts\ at\ 10\ am\ from\ A,$
$it\ will\ cross\ all\ launches\ that\ have\ started\ from\ b/$
$w\ 8:20\ am\ to\ 10:40\ am$
$\therefore It\ will\ cross\ below\ 8\ launches\ that\ have\ started\ from\ B$
$at\ following\ times:$
$8:20\ am, 8:40\ am, 9:00\ am, 9:20\ am, 9:40\ am, 10:00\ am,$
$10:20\ am\ \&\ 10:40\ am$