PRABHUTWA SPARDHA MODEL ANSWERS STD. 8 2024-2025

Q. 1] Let $4048 = x\,; 4047 = x - 1\,; 4049 = x + 1$

$$\therefore \frac{(4048^2 + 1)}{(4049^2 + 4047^2)} = \frac{x^2 + 1}{(x + 1)^2 + (x - 1)^2}$$

$$= \frac{x^2 + 1}{x^2 + 2x + 1 + x^2 - 2x + 1} = \frac{x^2 + 1}{2x^2 + 2} = \frac{x^2 + 1}{2(x^2 + 1)} = \frac{1}{2}$$

Q. 2] $5\frac{1}{x} = \frac{5x+1}{x}$ and $y\frac{3}{4} = \frac{4y+3}{4}$

$\therefore 5\frac{1}{x} \times y\frac{3}{4} = \left(\frac{5x + 1}{x}\right)\left(\frac{4y + 3}{4}\right) = 20$

$\therefore (5x + 1)(4y + 3) = 20 \times 4 \times x$

$\therefore (5x + 1)(4y + 3) = 20 \times 4x$

OR $(5x + 1)(4y + 3) = 40 \times 2x$

OR $(5x + 1)(4y + 3) = 16 \times 5x$ But x & y are positive intergers.

$\therefore\ (5x + 1)(4y + 3) = 16 \times 5x$

$\therefore\ (5x + 1) = 16\ \&\ (4y + 3) = 5x$

$\therefore\ x = 3\ \&\ y = 3$

Q. 3] $\sqrt{m\sqrt{m\sqrt{m}}} = 128$ Squaring both sides

$m \times \sqrt{m \times \sqrt{m}} = 128^2 = (2^7)^2 = 2^{14}$ Again squaring both sides

$m^2 m\sqrt{m} = (2^{14})^2 = 2^{28}$ Squaring again

$m^4 m^2 m = (2^{28})^2 = 2^{56}\ \therefore m^{4+2+1} = 2^{56} \therefore m^7 = 2^{56}$

$\therefore m^7 = (2^8)^7 \therefore m = 2^8 = 256$

Q. 4] $A = \pi r^2\ \therefore 34\pi = \pi r^2\ \therefore r = \sqrt{34}$

$\sqrt{34}$ is an irrational number & cannot be measured on scale.

Let's consider $3^2 + 5^2 = 34$

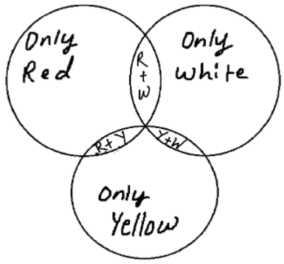
Hence, $\sqrt{34}$ is the hypotenuse of right-angled triangle of sides 3 cm & 5 cm.

Construct triangle ABC such that, angle B = 90^0, AB = 3 cm, BC = 5 cm, AC = $\sqrt{34}$ cm.

Draw a circle with C as the centre and AC as the radius.

This is the required circle with radius $\sqrt{34}$ & area 34π sq. cm.

Q. 5]



Total Area = A(Red) + A(White) + A(Yellow) − {A(R&W) + A(Y&W) + A(R&Y)}

∴ 100% = 70% + 40% + 30% − {A(R&W) + A(Y&W) + A(R&Y)}

∴ {A(R&W) + A(Y&W) + A(R&Y)} = 40%

Total Area = A(Only Red) + A(Only White) + A(Only Yellow) + {A(R&W) + A(Y&W) + A(R&Y)}

∴ 100% = A(Only Red) + A(Only White) + A(Only Yellow) + 40%

∴ A(Only Red) + A(Only White) + A(Only Yellow) = 100% − 40% = 60%

Q. 6] $E(1) + E(2) + E(3) + \cdots + E(100)$

$= \{E(1) + E(2) + \cdots + E(10)\} + \{E(11) + E(12) + \cdots E(20)\} + \cdots \{E(91) + E(92) + \cdots + E(100)\}$

$= \{0 + 2 + 0 + 4 + 0 + 6 + 0 + 8 + 0 + 0\} \times 10 + \{2 \times 10 + 4 \times 10 + 6 \times 10 + 8 \times 10\}$

$= \{2 + 4 + 6 + 8\} \times 20 = 400$

Q. 7] Let the original number be x ∴ New number is $x + 10$

∴ % increase $= \frac{\text{New} - \text{Original}}{\text{New}} \times 100$ ∴ % increase $= \frac{x+10-x}{x} \times 100$ ∴ % increase $= \frac{1000}{x}$

$\therefore 72 = (x + 10) + \left[\frac{1000}{x} \% \; of \; (x + 10)\right] \therefore 72 = (x + 10) + \left[\frac{10}{x}(x + 10)\right]$

$\therefore 72x = x(x + 10) + 10(x + 10) \;\therefore 72x = x^2 + 10x + 10x + 100$

$\therefore x^2 - 52x + 100 = 0 \;\therefore\; x^2 - 2x - 50x + 100 = 0 \;\therefore x(x - 2) - 50(x - 2) = 0$

$\therefore (x - 50)(x - 2) = 0 \;\therefore x = 50 \; OR \; x = 2$ ∴ the original number was 50 or 2.

Q. 8] Let CP of each article be x ∴ number of articles bought $= \frac{600}{x}$

SP $= x + 5$ ∴ Total Sale = SP(No. of articles sold) $= (x + 5)(\frac{600}{x} - 10) = \frac{-10x^2+550x+3000}{x}$

Total Profit $= \frac{-10x^2+550x+3000}{x} - 600 = \frac{-10x^2-50x+3000}{x}$

But, Total Profit = CP of 15 articles = $\frac{-10x^2-50x+3000}{x} = 15x$

$\therefore -10x^2 - 50x + 3000 = 15x^2 \;\therefore 25x^2 + 50x - 3000 = 0 \;\therefore x^2 + 2x - 120 = 0$

$\therefore (x + 12)(x - 10) = 0 \therefore x = -12 \; OR \; x = 10$ But $x \neq -12$

∴ CP of each article = Rs.10

Q. 9] $a = \frac{1}{2}(\sqrt{3}+1) \therefore 2a = \sqrt{3}+1 \quad \therefore 2a-1 = \sqrt{3} \therefore (2a-1)^2 = 3$

$\therefore 4a^2 - 4a - 2 = 0 \therefore 2a^2 - 2a = 1$

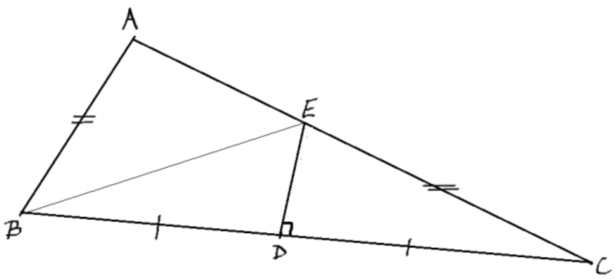
We have to find the value of $4a^3 + 2a^2 - 8a + 7$

$4a^3 + 2a^2 - 8a + 7 = 4a^3 - 4a^2 + 6a^2 - 2a - 6a + 7$

$= 4a^3 - 4a^2 - 2a + 6a^2 - 6a + 7 = a(4a^2 - 4a - 2) + 3(2a^2 - 2a) + 7$

$= a(0) + 3(1) + 7 = 10$

Q. 10]



Draw seg BE

$\Delta BED \cong \Delta CED$ (SAS test of congruence)

$\therefore$ seg BE = seg CE (corresponding sides of congruent triangles)

$\therefore \angle EBD \cong \angle ECD = x$ But $\angle ECD$ is same as $\angle ACB$. Hence, $\angle ACB = x$

$\angle AEB$ is exterior angle of ΔBEC $\therefore \angle AEB = 2x$

Since seg BE = seg CE & seg AB = Seg EC $\therefore$ seg BE $\cong$ segAB

$\therefore \Delta AEB$ is isosceles & $\angle BAC \cong \angle AEB = 2x$

In $\Delta ABC, \angle ABC + \angle BAC + \angle ACB = 180°$

$\therefore 57° + 2x + x = 180° \therefore 3x = 180 - 57 = 123 \therefore x = 41° \therefore \angle ACD = 41°$

Q. 11] Let a,b,c be the time required by taps A, B & C respectively to fill the entire tank.

$\frac{1}{a} \times t + \frac{1}{b} \times 10 + \frac{1}{c} \times (t-12) = 1$ (eq.1) & $\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$ (eq. 2)

As each tap fills equal volume of the tank, it means each tap fills $1/3^{rd}$ volume.

$\therefore \frac{t}{a} = \frac{10}{b} = \frac{t-12}{c} = \frac{1}{3}$ (eq. 3)

Substituting b = 30 in eq. 2, $\frac{1}{a} + \frac{1}{30} = \frac{1}{t} \therefore \frac{t}{a} = 1 - \frac{t}{30}$

But $\frac{t}{a} = \frac{1}{3} \therefore 1 - \frac{t}{30} = \frac{1}{3} \therefore t = 20$. Using eq.3, $\frac{t-12}{c} = \frac{1}{3} \therefore c = 24$ minutes

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Q. 12] Let $x = a \times h$ and $y = b \times h$ where h is HCF of x and y

Hence, a & b are co-prime numbers and $x + y = 1050$ (Given)

$\therefore ah + bh = 1050 \;\; \therefore 1050 = h(a + b)$

$\therefore 2 \times 3 \times 5 \times 5 \times 7 = h(a + b)$

Since h must be maximum, $a + b$ should be minimum.

$If\ a + b = 2, a = 1\ \&\ b = 1 \quad i \cdot e \cdot x = y$

But x & y are distinct. Hence, $a + b \neq 2$.

$If\ a + b = 3, a = 1$ and $b = 2$.

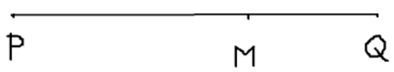
Then $h = 2 \times 5 \times 5 \times 7 = 350$

$\therefore x = 1 \times 350$ and $y = 2 \times 350$

$\therefore h = 350\ ; x = 350\ ; y = 700$

$\therefore$ The numbers are 350 and 700.

Q. 13]



Let M be the point of meet; Let S_c & S_t be the speeds of car & truck respectively.

Car travels P to Q from 9: 00 am to 1: 00 pm, i. e in 4 hrs

$\therefore PQ = S_c \times T = 90 \times 4 = 360$ km;

$QM = \frac{1}{4} \times 360 = 90$ km;

$PM = 360 - 90 = 270$ km

$\therefore$ time taken by car to reach pt. M $= \frac{QM}{S_c} = \frac{270}{90} = 3$ hrs,

i. e. 3 hrs after 9: 00am = at 12: 00 pm

Truck covers distance QM from 10: 00am till 12: 00pm i. e. in 2 hrs

$\therefore S_t = \frac{QM}{Time} = \frac{90}{2} = 45$ km/h

$\therefore$ Time taken by truck to cover QP $= \frac{QP}{S_t} = \frac{360}{45} = 8$ hrs

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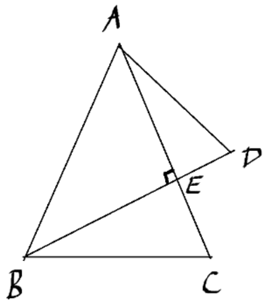
Q. 14]



Let $\angle BAC = x$ & $\angle ABD = y$

As ΔABC is isosceles, $\angle ABC = \angle ACB = \frac{180 - x}{2}$

As ΔBAD is isosceles, $\angle BAD = \angle ADB = \frac{180 - y}{2}$

In ΔAEB, $\angle AEB + \angle ABD + \angle BAC = 180 \therefore 90 + y + x = 180 \therefore x + y = 90$

$\angle ACB + \angle ADB = \frac{180-x}{2} + \frac{180-y}{2}$

$\therefore \angle ACB + \angle ADB = 180 - \frac{x+y}{2} = 180 - \frac{90}{2} = 135$

Q. 15] Avg marks of 10 students $= \frac{\text{Total marks of 10 students}}{\text{No.of students}}$

$\therefore$ Total marks of 10 students $= 60 \times 10 = 600$

$\therefore$ Avg marks of bottom 5 students $= 60 - 5 = 55$

$\therefore$ Avg marks of bottom 5 students $= \frac{\text{Total marks of bottom 5 students}}{5}$

$\therefore$ Total marks of bottom 5 students $= 55 \times 5 = 275$

$\therefore$ Total marks of top 5 students

= Total marks of 10 students − Total marks of bottom 5 students

$\therefore$ Total marks of top 5 students $= 600 - 275 = 325$

$\therefore$ Let the distinct scores of top 5 students be a, b, c, d, e

$\therefore a + b + c + d + e = 325$

$\therefore$ If a must be maximum, b, c, d, e should be as low as possible

Also, 6th ranker's score $<$ e and the average of bottom 5 rankers is 55

$\therefore$ As e is as low as possible, it means 6th ranker's score is also as low as possible & avg is 55

In bottom 5, if one scores less than 55, other must score more than 55 to maintain the average at 55

$\therefore$ All the bottom 5 should score 55 marks, so that 6th ranker's score is low

$\therefore e = 56, d = 57, c = 58, b = 59 \therefore a + b + c + d + e = 325$

$\therefore a = 325 - (56 + 57 + 58 + 59) \therefore a = 95$

So, the maximum possible marks of the topper are 95
